STANDING WAVES FOR A GENERALIZED DAVEY–STEWARTSON SYSTEM: REVISITED

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Abstract. The existence of standing waves for a generalized Davey–Stewartson (GDS) system was shown in Eden and Erbay [8] using an unconstrainted minimization problem. Here, we consider the same problem but relax the condition on the parameters to $\chi+b<0$ or $\chi+\frac{b}{m_1}<0$. Our approach, in the spirit of Berestycki, Gallouët and Kavian [3] and Cipolatti [6], is to use a constrained minimization problem and utilize Lions' concentration-compactness theorem [11]. When both methods apply we show that they give the same minimizer and obtain a sharp bound for a Gagliardo–Nirenberg type inequality. As in [8], this leads to a global existence result for small-mass solutions. Moreover, following an argument in Eden, Erbay and Muslu [9] we show that when $p>2$, the L^p-norms of solutions to the Cauchy problem for a GDS system converge to zero as $t\to\infty$.

1. Introduction

The existence of standing waves for a GDS system was established in [8] by extending the analysis done by Weinstein for the NLS equation [13] and by Papanicolaou et. al. for the DS system [12]. In this note, our aim is to follow a different route and obtain the existence of standing waves for a GDS system under less stringent conditions on the parameters. Our interest lies in $n=2$ case and the relevant work for the NLS was done by Weinstein [13] and Berestycki, Gallouët and Kavian [3] where in the latter in addition to the existence of ground states the existence of infinitely many solutions was also established. Later, Cipolatti showed the existence of standing waves for the DS system when $n=2$ or 3 [6]. Our aim is to modify these arguments so that they apply to a larger class of equations that include the GDS system as a special case. Here, however, due to assumption (A3) we are not treating the more general case considered in [8].

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The GDS system was derived by Babaoglu and Erbay [2] to model the propagation of waves in a bulk medium composed of an elastic medium with couple stresses. In [1] it was classified as elliptic–elliptic–elliptic(EEE), elliptic–hyperbolic–hyperbolic and elliptic–elliptic–hyperbolic depending on the signs of the physical parameters. There some results on the global existence and non-existence were obtained in the EEE case. This is also the case we will consider here. In [7] the problem of existence of travelling waves for GDS system was considered for the cases EEE and HEE. The necessary conditions for existence were Pohozaev type identities. Later in [8] Pohozaev type identities played an important role in restricting the parameters ω, χ and b in order to establish the existence of standing waves. Pohozaev identities for solutions can be derived in different ways and here we choose an alternative approach.

Our paper is organized as follows: in the second section we summarize the results obtained in [8] leading to the existence of standing waves paying special attention on the gap between the necessary conditions for existence and the sufficient conditions that are actually imposed. Weinstein's approach in [13] is to minimize a non-linear functional J over $H^1(\mathbb{R}^2)$. Here care is needed in order to avoid the denominator of J being zero. Sufficient conditions that are imposed in [8] serve this purpose. In contrast, in an alternative approach, when $n = 2$ the kinetic energy is minimized over a space where potential energy is zero [3, 6]. The two types of energies have different behaviour under different scaling transformations, these are summarized in the third section. Next we state our main theorem on the existence of standing waves followed by a remark where we show that whenever both methods apply they result in the same solutions. At the end of that section, in harmony with the scaling transformations, we indicate alternative proofs for Pohozaev type identities. In the forth section we prove a Gagliardo–Nirenberg type inequality and establish global existence of solutions of the GDS system. Moreover we show that these solutions tend to zero in L^p for $p > 2$ as $t \to \infty$. We conclude with a comparison of two methods by showing that the present method works for the GDS under the weaker assumption $\chi + b < 0$ or $\chi + \frac{b}{m_1} < 0$.

Throughout this paper $\| \cdot \|_p$ will denote the L^p-norm for $1 \leqslant p < \infty$, whereas we will write $\| \cdot \|_{W^{m,p}}$ for Sobolev space norms. Also (f, g) will denote $\int fg$ over $\mathbb{R}^2$.

2. Review of previous results

The equations introduced in [2] can be written in the EEE case as a cubic NLS equation with an additional non-local term in two space dimensions:

$$iv_t + \Delta v = \chi |v|^2 v + bK(|v|^2)v, \tag{1}$$

where the non-local term is given in terms of Fourier transform variables $\xi = (\xi_1, \xi_2)$ as $\widehat{K(f)}(\xi) = \alpha(\xi)\widehat{f}(\xi)$ with

$$\alpha(\xi) = \frac{\lambda\xi_1^4 + (1 + m_1 - 2n)\xi_1^2\xi_2^2 + m_2\xi_2^4}{\lambda\xi_1^4 + (m_1 + \lambda m_2 - n^2)\xi_1^2\xi_2^2 + m_1 m_2\xi_2^4}. \tag{2}$$

The symbol $\alpha(\xi)$ then satisfies:

(A1) $\alpha(\xi)$ is even and homogenous of degree zero,
(A2) $0 \leqslant \alpha(\xi) \leqslant \alpha_M$ for all $\xi \in \mathbb{R}^2$,
(A3) $\alpha_1 := \lim_{s\to\infty} \alpha(s\xi_1, \xi_2)$ and $\alpha_2 := \lim_{s\to 0^+} \alpha(s\xi_1, \xi_2)$ exist,

where for the GDS system $\alpha_M = \max\{1, 1/m_1\}$ [1] and $\alpha_1 = 1$, $\alpha_2 = 1/m_1$. In this paper, we will only assume that the symbol $\alpha(\xi)$ satisfies (A1)-(A3) hence our results will apply to the GDS system. For $v_0 \in H^1(\mathbb{R}^2)$ the existence and uniqueness of solutions to the Cauchy problem for the GDS system was discussed in [1]. Moreover it was shown that the Hamiltonian

$$H(v) = \int_{\mathbb{R}^2} \left(|\xi|^2|\widehat{v}|^2 + \frac{1}{2}(\chi + b\alpha(\xi)) \left|\widehat{|v|^2}\right|^2 \right) d\xi \tag{3}$$

for the GDS system is conserved in the EEE case. It can easily be checked that the same quantity is conserved for solutions of (1) under (A1) and (A2) [10].

Looking for a solitary wave in (1) of standing wave type, that is, v is of the form $e^{i\omega t}u(x)$ with $u \in H^1(\mathbb{R}^2)$, one is led to the equation

$$-\Delta u + \omega u = -\chi |u|^2 u - bK(|u|^2)u. \tag{4}$$

One of the key properties of the map K is that $K : L^p(\mathbb{R}^2) \to L^p(\mathbb{R}^2)$ is bounded for all $1 < p < \infty$ and $\|K(f)\| \leqslant \alpha_M \|f\|_2^2$. This and further properties of K are given in [8, Lemma 2.1]. Also we know that if u is a solution of (4), then $u \in \bigcap_{m=1}^{\infty} W^{m,p}$ for all $2 \leqslant p < \infty$ and there exist positive constants C, ν such that $|u(x)| + |\nabla u(x)| \leqslant Ce^{-\nu|x|}$ for all $x \in \mathbb{R}^2$ and $\lim_{|x|\to\infty} K(|u|^2)(x) = 0$ [8, Lemma 2.2]. Here we remark that we can take $\omega = 1$ without loss of generality by defining ψ as $u(x) = \sqrt{\omega}\psi(\sqrt{\omega}x)$.

In [8, Theorem 2.1], the following necessary conditions were obtained for the solutions of (4):

$$\int_{\mathbb{R}^2} (|\nabla R|^2 - \omega R^2) dx = 0, \qquad \int_{\mathbb{R}^2} (2\omega + \chi R^2 + bK(R^2)) R^2 dx = 0. \tag{5}$$

From (5) the two inequalities $\omega > 0$ and $\chi\|R\|_4^4 + b(K(R^2), R^2) < 0$ followed as necessary conditions on the solutions. To guarantee the latter inequality it was assumed that $\chi < \min\{-b\alpha_M, 0\}$. This is no longer assumed in this paper and we relax it (in Theorem 1) to $\chi + \alpha_1 b < 0$ or $\chi + \alpha_2 b < 0$. In [8] under the assumption $\chi < \min\{-b\alpha_M, 0\}$, the functional

$$J(f) = \frac{-2\|f\|_2^2 \|\nabla f\|_2^2}{\chi\|f\|_4^4 + b(K(|f|^2), |f|^2)}$$

was shown to have minimum on $H^1(\mathbb{R}^2)$, say R, which then satisfies (4) after a proper normalization, hence the following Gagliardo–Nirenberg type inequality was obtained as a corollary to [8, Theorem 2.1]:

$$-\chi\|f\|_4^4 - b(K(|f|^2), |f|^2) \leqslant C_{\text{opt}} \|f\|_2^2 \|\nabla f\|_2^2, \tag{6}$$

where $C_{\text{opt}} = 2/\|R\|_2^2$.

Now we will adapt the approach of Berestycki and Lions [4] and Berestycki, Gallouët and Kavian [3] for the NLS equation and consider a constrained minimization problem.

3. Existence of Standing Waves

We note that $u \neq 0$ solves (4) if and only if u is a critical point of the Lagrangian given by

$$L_\omega(u) = \frac{1}{2}\|\nabla u\|_2^2 + \frac{b}{4}B(|u|^2) + \frac{\chi}{4}\|u\|_4^4 + \frac{\omega}{2}\|u\|_2^2,$$

where $B(f) := \int \alpha(\xi)|\widehat{f}(\xi)|^2 d\xi = \int K(f)(x)\overline{f(x)}dx$.

Various parts of this Lagrangian are invariant under different scalings [8]: if

$$u_{a,b}(x) := s^a u(s^b x), \quad \text{for some } s > 0, \tag{7}$$

then we have

$$\begin{aligned} \|u_{a,b}\|_2^2 &= s^{2a-2b}\|u\|_2^2, & \|\nabla u_{a,b}\|_2^2 &= s^{2a}\|\nabla u\|_2^2, \\ \|u_{a,b}\|_4^4 &= s^{4a-2b}\|u\|_4^4, & B(|u_{a,b}|^2) &= s^{4a-2b}B(|u|^2). \end{aligned} \tag{8}$$

There is also a partial scaling that reveals the closer kinship between $B(|u|^2)$ and $\|u\|_4^4$. Letting

$$u_s(x) = u_s(x_1, x_2) = s^{1/4}u(sx_1, x_2), \tag{9}$$

we get $B(|u_s|^2) = \int \alpha(s\xi_1, \xi_2)\left|\widehat{(|u|^2)}(\xi_1, \xi_2)\right|^2 d\xi$. By (A3) and Lebesgue dominated convergence theorem it follows that $\lim_{s\to\infty} B(|u_s|^2) = \alpha_1\|u\|_4^4$ and $\lim_{s\to 0^+} B(|u_s|^2) = \alpha_2\|u\|_4^4$.

Using the standard terminology, as in [5, 6], we set

$$T(u) := \|\nabla u\|_2^2, \qquad V(u) := -\frac{b}{4}B(|u|^2) - \frac{\chi}{4}\|u\|_4^4 - \frac{\omega}{2}\|u\|_2^2$$

so that $L_\omega(u) = \frac{1}{2}T(u) - V(u)$ is to be minimized over $H^1(\mathbb{R}^2)$. To fix some notation, define $\Sigma_0 := \{u \in H^1(\mathbb{R}^2) : u \neq 0, V(u) = 0\}$ and $I := \inf\left\{\frac{1}{2}T(u) : u \in \Sigma_0\right\}$. Then it can be easily shown that if $\Sigma_0 \neq \emptyset$ and $\omega > 0$ then $I > 0$.

Theorem 1. *For $\chi + \alpha_1 b < 0$ or $\chi + \alpha_2 b < 0$, and $\omega > 0$ the minimization problem*

$$\begin{gathered} u \in \Sigma_0, \\ T(u) = \min\{T(\psi) : \psi \in \Sigma_0\} = 2I, \end{gathered} \tag{10}$$

has a positive solution. This solution satisfies $0 < L_\omega(u) \leqslant L_\omega(\psi)$ among all $\psi \in H^1(\mathbb{R}^2)$ solving (4). Moreover, if u is properly scaled then it is a solution of (4).

Proof. First we will note that Σ_0 is not empty. To establish this we will use one parameter scalings introduced in (7) and (9). If $\chi + \alpha_1 b < 0$, for $u \in H^1(\mathbb{R}^2)$ defining u_s as in (9), $s \to \infty$ implies $(-bB(|u_s|^2) - \chi\|u_s\|_4^4) \longrightarrow -(\chi + \alpha_1 b)\|u\|_4^4 > 0$. Thus there exists s_0 large enough such that $-bB(|u_{s_0}|^2) - \chi\|u_{s_0}\|_4^4 > 0$. Considering $V(su_{s_0})$, a quintic polynomial in s, as the leading coefficient is positive there exists an s_1 so that $V(s_1 u_{s_0}) = 0$. Similarly if $\chi + \alpha_2 b < 0$ we send $s \to 0^+$ to have $(-bB(|u_s|^2) - \chi\|u_s\|_4^4) \longrightarrow -(\chi + \alpha_2 b)\|u\|_4^4 > 0$, hence, we choose s_0 close to 0 such that $-bB(|u_{s_0}|^2) - \chi\|u_{s_0}\|_4^4 > 0$. Rest of the argument follows as above.

Now, let $(u_n) \subset \Sigma_0$ be a minimizing sequence such that $\|u_n\|_2 = 1$. Since $T(u_n)$ is bounded so is $\|u_n\|_{H^1}$, hence there exists $u \in H^1(\mathbb{R}^2)$ and a subsequence such that $u_n \rightharpoonup u$ weakly in H^1. In order to utilize the concentration compactness principle of Lions [11] we consider

$$\rho_n(x) = |\nabla u_n(x)|^2 + |u_n(x)|^2,$$

where $\int_{\mathbb{R}^2} \rho_n(x)dx = T(u_n) + \|u_n\|_2^2 \to 2I + 1$. There are three possibilities: vanishing, dichotomy or concentration. Since concentration is the only possibility that occurs, there exists $(y_n) \subset \mathbb{R}^2$ such that for every $\epsilon > 0$, there exists $R_\epsilon \geqslant \frac{1}{\epsilon}$ and

$$\int_{\mathbb{R}^2 \setminus B_{R_\epsilon}(y_n)} \rho_n(x)dx \leqslant \epsilon.$$

Replacing $u_n(x)$ by $\widetilde{u_n}(x) = u_n(x - y_n)$, $\widetilde{u_n} \rightharpoonup \widetilde{u}$ weakly in $H^1(\mathbb{R}^2)$ and by the imbedding $H^1(\mathbb{R}^2) \hookrightarrow L^p(\mathbb{R}^2)$ for $2 \leqslant p < \infty$, it follows that $\int_{\mathbb{R}^2 \setminus B_{R_\epsilon}(0)} |\widetilde{\varphi_n}|^2 dx \leqslant \epsilon^{p/2}$ for $2 \leqslant p < \infty$. Over $B_{R_\epsilon}(0)$ the imbedding is compact and we can pass to the limit in V. Combining these two, from $V(\widetilde{u_n}) = 0$ it follows that $V(\widetilde{u}) = 0$, i.e., $\widetilde{u} \in \Sigma_0$ with $T(\widetilde{\varphi}) \leqslant \liminf_{n\to\infty} T(\widetilde{\varphi_n}) = 2I$. Hence $\widetilde{u}$ is the desired minimum. Positivity of this minimum follows from [5, Lemma 8.1.12]. If u solves the minimization problem and ψ is any solution of (4) then from the Pohozaev like identities in [8], we get that $V(\psi) = 0$, hence, $L_\omega(u) \leqslant L_\omega(\psi)$.

Let u be a solution of (10). Then there is a Lagrange multiplier s such that $-\Delta u = s(-bK(|u|^2)u - \chi|u|^2 u - \omega u)$, where $s > 0$ can be shown. From that we have a solution of (4) under the scaling $u_{0,-1/2} = u(x/\sqrt{s})$. □

Remark 1. The minimum of T does not change if we replace Σ_0 by $\{u \in H^1(\mathbb{R}^2) : u \neq 0, V(u) \geqslant 0\}$. This is easy to see using one parameter scalings defined in (7),i.e., the fact that if $V(u) \geqslant 0$ then there exists $0 < s \leqslant 1$ such that $V(su) = 0$.

Remark 2. Here we want to highlight that minimizers obtained from both methods coincide. From Theorem 2.2 in [8], there exists R, which minimizes $J = \frac{-2\|f\|_2^2\|\nabla f\|_2^2}{\chi\|f\|_4^4 + bB(|f|^2)}$ over H^1. Also R satisfies Pohozaev type identities, i.e., $T(R) = \omega\|R\|_2^2$ and $V(R) = 0$. Noting that for any u with $V(u) = 0$, $J(u) = \frac{1}{\omega}T(u)$ and hence $\frac{1}{\omega}T(R) \leqslant J(\psi)$ for all $\psi \in H^1$. Restricting this inequality to Σ_0 we see that R minimizes T over Σ_0. Conversely, let $u \in \Sigma_0$ be a minimizer of T and let $\psi \in H^1$. If $V(\psi) = 0$, clearly $J(u) \leqslant J(\psi)$. Otherwise consider $V(s\psi)$. Since $\chi < \min\{-b\alpha_M, 0\}$, there exists s_0 such that $V(s_0\psi) = 0$. Note that $J(\psi) = J(s_0\psi)$, hence we get that $J(u) \leqslant J(s_0\psi) = J(\psi)$ and so u is a minimizer of J over H^1.

Here we want to outline how to establish Pohozaev type identities given in [8] in an alternative way.

Proposition 1. If $u \in H^1$ is a solution of *(4)* then

$$T(u) + \omega\|u\|_2^2 = -bB(|u|^2) - \chi\|u\|_4^4, \qquad 2\omega\|u\|_2^2 = -bB(|u|^2) - \chi\|u\|_4^4.$$

Proof. Note that if u is a solution of (4) then it is a critical point of L_ω. To show the first identity, differentiate L_ω along the one parameter family defined by $s \longmapsto u_{1,0}$. Since $L_\omega(u_{1,0}) = s^2\frac{1}{2}T(u)+s^4\frac{b}{4}B(|u|^2)+s^4\frac{\chi}{4}\|u\|_4^4+s^2\frac{\omega}{2}\|u\|_2^2$, the result follows from $\left.\frac{dL_\omega(u_{1,0})}{ds}\right|_{s=1} = 0$. For the second identity, differentiate L_ω along $s \longmapsto u_{0,-1}$. Using the scalings given in (8), $L_\omega(u_{0,-1}) = \frac{1}{2}T(u)-\lambda^2V(u)$. Hence $\left.\frac{dL_\omega(u_{0,-1})}{ds}\right|_{s=1} = 0$ yields the second identity. □

4. A Gagliardo–Nirenberg Type Inequality and its Consequences

One of the contributions of this paper is an alternative derivation of the Gagliardo–Nirenberg type inequality using the constrained minimization problem described in the previous section. When $\chi + \alpha_1 b < 0$ or $\chi + \alpha_2 b < 0$, in the unconstrained minimization problem (see Section 2) the denominator of the functional J can become zero for $u \in H^1(\mathbb{R}^2)$, hence this method does not seem to be applicable. On the other hand, in the constrained minimization problem the potential $V(u)$ can be made to change sign through a continuous one parameter family of functions passing from u. This fact plays an important role in the derivation of the main result of this section.

Theorem 2. *If $\chi + \alpha_1 b < 0$ or $\chi + \alpha_2 b < 0$ for any $f \in H^1(\mathbb{R}^2)$ we have*

$$-\left(\chi\|f\|_4^4 + bB(|f|^2)\right) \leqslant \frac{\omega}{I}\|f\|_2^2\|\nabla f\|_2^2,$$

where $I = \frac{1}{2}T(u)$ and u is a solution of (4).

Proof. Let $f \in H^1(\mathbb{R}^2)$ be arbitrary. First, if $V(f) = 0$ then we know that $I \leqslant \frac{1}{2}\|\nabla f\|_2^2$. Hence we establish the result. Second, assume $V(f) > 0$. Since $\omega > 0$ we have $-\chi\|f\|_4^4 - bB(|f|^2) > 0$ hence using scaling properties of V we can show the existence of an s such that $V(sf) = 0$. Since J is invariant under these type of scalings the result follows from the first case. Finally, if $V(f) < 0$ the result follows trivially when $-\chi\|f\|_4^4 - bB(|f|^2) \leqslant 0$. If $V(f) < 0$ but $-\chi\|f\|_4^4 - bB(|f|^2) > 0$, considering $V(sf)$ as a quintic polynomial as before we can find $s_0 > 1$ so that $V(s_0 f) = 0$ hence the first case applies. □

Remark 3. The connection between I and C_{opt}, where C_{opt} is given in (6), is established as follows: For R obtained in [8, Theorem 2.2], we have $\frac{1}{\omega}T(R) \leqslant \frac{1}{\omega}T(u)$ for all $u \in \Sigma_0$. Hence $\frac{1}{\omega}T(R) \leqslant \frac{1}{\omega}\inf\{T(u) : u \in \Sigma_0\} = \frac{2I}{\omega}$. Since $R \in \Sigma_0$ from the Pohozaev type identities, $\inf T(u) \leqslant T(R)$. Noting that $T(R) = \omega\|R\|_2^2$ we have $\frac{\omega}{C_{\mathrm{opt}}} = \frac{\omega}{2}\|R\|_2^2 = \frac{1}{2}T(R) = I$.

Using this estimate we can find an upper bound on the initial condition and hence state the following global existence result whose proof follows as in [8].

Corollary 1. *For the Cauchy problem for the GDS system, if $\chi + b < 0$ or $\chi + \frac{b}{m_1} < 0$, and $\|v_0\|_2 < \|u\|_2$, where $v_0 \in H^1(\mathbb{R}^2)$ is the initial amplitude and u is a solution of (4), then the corresponding solution of the GDS system is global.*

Also the asymptotic behaviour of solutions follows as a corollary:

Corollary 2. *Let v be a solution to the Cauchy problem for a GDS system and assume that v remains in $\Sigma := \{v \in H^1(\mathbb{R}^2) : (x^2+y^2)^{1/2},\, v \in L^2(\mathbb{R}^2)\}$. If $\chi+b<0$ or $\chi+\frac{b}{m_1}<0$, and $\|v_0\|_2 < \|u\|_2$, where u is a solution of (4), then*

$$\|v(t)\|_p^p \leqslant C(1+|t|)^{2-p},$$

for $t>0$, $p>2$ where C depends only on v_0 and p.

Proof. In fact, $\|v_0\|_2 < \|u\|_2$ implies that $\|\nabla v(t)\|_2^2 \leqslant MH(v_0)$ for every $t>0$, with $M = \left(1 - \frac{\|v_0\|_2^2}{\|u\|_2^2}\right)^{-1}$. Proceeding as in [9, Section 4] the result follows. □

In order to adapt the argument in [9] to the present situation one needs the validity of the pseudoconformal invariance under (A1) and (A2). This is addressed in Eden and Kuz [10] as well as the existence and uniqueness for the Cauchy problem for (4) under (A1) and (A2).

5. Conclusion

The hypothesis (A3) is satisfied by the symbol of DS system with $\alpha_1=\alpha_2=1$ and by the symbol of the GDS system with $\alpha_1=1$ and $\alpha_2=\frac{1}{m_1}$. (A3) was not assumed in [8], hence in a certain sense the result in [8] on existence is more general. However, (A3) plays the key role in the scaling $u \leftrightarrow u_s$ defined in (9) and in the relation between $B(|u|^2)$ and $\|u\|_4^4$. (A3) is our first attempt to obtain the partial scaling given in (9), there might be other types of partial scalings that will also work.

Under the dilation $u \leftrightarrow su$, J is invariant whereas $V(su)$ can be made equal to zero when $\chi+\alpha_1 b<0$ or $\chi+\alpha_2 b<0$. Note that, although J is invariant under the scalings $u \leftrightarrow u_{a,b}$ defined in (7), it is no longer invariant under the partial scaling (9) $u \leftrightarrow u_s$.

Comparing the condition $\chi < \min\{-b\alpha_M, 0\}$ with $\chi+b<0$ or $\chi+\frac{b}{m_1}<0$ for the GDS system, we see that, when $b>0$, the first condition reduces to $\chi+b\alpha_M<0$. Since $\alpha_M \geqslant 1$ and $\alpha_M \geqslant \frac{1}{m_1}$ this is a stronger assumption than $\chi+b<0$ or $\chi+\frac{b}{m_1}<0$. When on the other hand $b<0$, from the first condition we have $\chi<0$, whereas $\chi<-b$ or $\chi<-\frac{b}{m_1}$ allows positive values for χ as well. When $m_1=1$, hence $\alpha_M=1$, there is still improvement in $\chi+b<0$ case.

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